An Inhomogeneous Spatial Node Distribution and its Stochastic Properties

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ABSTRACT

Most analysis and simulation of wireless systems assumes that the nodes are randomly located, sampled from a uniform distribution. Although in many real-world scenarios the nodes are non-uniformly distributed, the research community lacks a common approach to generate such inhomogeneities. This paper intends to go a step in this direction. We present an algorithm to create a random inhomogeneous node distribution based on a simple neighborhood-dependent thinning of a homogeneous Poisson process. We derive some useful stochastic properties of the resulting distribution (in particular the probability density of the nearest neighbor distance) and offer a reference implementation. Our goal is to enable fellow researchers to easily use inhomogeneous distributions with well-defined stochastic properties.

Categories and Subject Descriptors

C.2.1 [**Network architecture and design**]: Wireless communication; G.3 [**Probability and statistics**]: Stochastic processes; I.6.5 [**Model Development**]: Modeling methodologies

General Terms

Design, Performance, Theory

1. INTRODUCTION

Performance analysis of wireless networks requires a set of modeling assumptions which describe — in a simplified manner — the behavior of the system and its environment. An important modeling issue is the spatial distribution of users and devices. Unfortunately, real-world location data from wireless systems is difficult to obtain from operators and often has a low spatial resolution.

Hence, a common approach is to employ a "synthetic" model for the node distribution, where nodes are placed randomly in a system area. The vast majority of publications uses such a model. Typically, a *uniform (= homogeneous)* random node distribution over the system area is employed (see Fig. 1(a)). In some cases, however, researchers want to use a *non-uniform* (= *inhomogeneous*) random distribution, modeling the fact that nodes are concentrated in hotspots, such as buildings, malls, and pedestrian zones (see Fig. 1(c)). Despite this demand, a commonly accepted approach to model such inhomogeneities is missing in our research community. In particular, there is no model of which stochastic properties are well-known.

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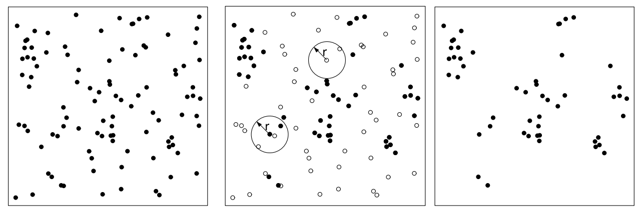


(a) Uniform (b) Thinning (c) Non-uniform

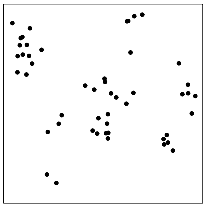
Figure 1: Spatial distributions and thinning

The goal of this paper is to remove this lack. We present a method that creates inhomogeneous stochastic node distributions based on the concept of "thinning" known in spatial statistics [1, 2]. This approach is flexible, yet simple to implement and not restricted to certain scenarios. We derive some important stochastic properties of the resulting distributions and offer a reference implementation. By doing so, we try to contribute a new building block to the simulation and analysis of wireless systems.

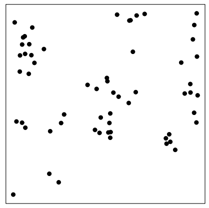
2. INHOMOGENEOUS DISTRIBUTION

Our method can generate an inhomogeneous random node distribution for both bounded and infinitely-large areas. The generation of a inhomogeneous distribution on a **bounded area** involves two steps: First, we generate a uniform random node distribution $\mathcal{U}$, i.e., we place m nodes i.i.d. on a given area A, where the location of each node is sampled from a uniform random distribution over the area. Second, we remove some of the nodes according to a certain algorithm. This "thinning" of $\mathcal{U}$ yields a non-uniform node distribution $\mathcal{I}$.

The thinning algorithm can be accomplished as follows: For each node, we check the number of nodes located within a circle of radius r around its position. The nodes within this circle are called the node's "neighbors." A node is marked to be thinned if it has less than k neighbors. After each node has been classified, those nodes marked to be thinned are actually removed. Alternatively, we can think of a node to be retained if its kth nearest neighbor is at most a distance of r away. Note that the algorithm does not depend on the order in which the nodes are processed.



(a) Normal distance



(b) Wrap-around

Figure 2: Border effect in the thinning algorithm

An example is illustrated in Fig. 1, where $m = 100$ initial nodes are uniformly distributed on a square area with side length $\sqrt{A} = 5$ length units. Fig. 1(b) shows the neighborhood circle of radius $r = 0.5$ length unit of two nodes. Nodes are removed if they have less than $k = 3$ neighbors within this circle. The nodes shown as solid dots will survive, and the nodes shown as circles will be removed. In this example, there are $m' = 50$ remaining nodes.

This thinning algorithm suffers from a **border effect**: Nodes close to the border of A have, on average, less neighbors and are therefore more likely to be removed. This effect is similar to the border effect occurring in the analysis of connectivity in ad hoc networks [3, 4]. To avoid this effect, a wrap-around distance measure can be used in place of the standard Euclidean distance [1]. A node at the border of A is considered to be close to a node at the opposite border of A, hence the two nodes can be neighbors.

Fig. 2 demonstrates the border effect using the same initial distribution as above. Clearly, in the wrap-around case, more nodes located close to the border survive the thinning. In this example, there are $m' = 60$ remaining nodes.

The same thinning can be applied to nodes distributed on an **infinite area** according to a homogeneous Poisson point process. In fact, a homogeneous distribution $\mathcal{H}$ generated by a Poisson process can be regarded as the limiting case of a uniform distribution $\mathcal{U}$ in which $m \to \infty$ and $A \to \infty$ but the node density $\rho = m/A$ remains constant. If we thin out $\mathcal{H}$ and regard a subarea of size A', stochastically essentially the same node pattern will result as if thinning was applied to $\mathcal{U}$ without border effects.

3. STOCHASTIC PROPERTIES

This section derives some stochastic properties of inhomogeneous distributions $\mathcal{I}$ when being generated without border effects. Among other things, we derive the percentage of nodes remaining and the probability density function (pdf) of the distance of a node to its nearest neighbor in $\mathcal{I}$. The latter parameter is an essential distance property that has significant impact on network connectivity [4]. Results are derived assuming a Poisson process generating the underlying homogeneous distribution $\mathcal{H}$, but the results also apply to bounded areas with a wrap-around distance model.

3.1 Percentage of Nodes Remaining

The number of neighbors of a node N is denoted by the random variable K. Per definition of a Poisson process, K follows a Poisson distribution, i.e., $\mathrm{P}(K = i) = P_\mu(i) := \mathrm{e}^{-\mu}\, \mu^i/i!$. The constant μ represents the expected number of neighbors, i.e., $\mu := \rho r^2 \pi$. The probability for a node N to be removed is thus given by $\mathrm{P}(N\,\text{removed}) = \mathrm{P}(K < k) = \sum_{i=0}^{k-1} \mathrm{P}(K = i)$. Using the incomplete Gamma function $\Gamma(k,\mu) = (k-1)!\ \mathrm{e}^{-\mu} \sum_{i=0}^{k-1} \mu^i/i!$, we obtain

$$\mathrm{P}(N\,\text{removed}) = \frac{\Gamma(k,\mu)}{(k-1)!}\,. \tag{1}$$

The probability that a node N is not removed, i.e., it survives the thinning is

$$\mathrm{P}(N\,\text{survives}) = 1 - \mathrm{P}(N\,\text{removed}) = 1 - \frac{\Gamma(k,\mu)}{(k-1)!}\,. \tag{2}$$

The expected number of nodes remaining in an area of size A is thus

$$\mathrm{E}(m') = \rho\, A\, \mathrm{P}(N\,\text{survives}) \;=\; \rho\, A \left(1 - \frac{\Gamma(k, \rho\, r^2 \pi)}{(k-1)!}\right)\,. \tag{3}$$

3.2 Number of Previous Neighbors

We consider a node N_0 that survives the thinning operation and are interested in the number of neighbors K_0 that this node had in the original distribution $\mathcal{H}$. This number is no longer Poisson distributed due to the condition that N_0 survives. For $i \geq k$, we can write

$$\begin{aligned}\mathrm{P}(K_0 = i \mid N_0\,\text{survives}) &= \mathrm{P}(K_0 = i \mid K_0 \geq k)\\ &= \frac{\mathrm{P}(K_0 = i \wedge K_0 \geq k)}{\mathrm{P}(K_0 \geq k)} = \frac{P_\mu(i)}{1 - \frac{\Gamma(k,\mu)}{(k-1)!}}\,.\end{aligned} \tag{4}$$

The expected number of (previous) neighbors of a surviving node is $\mathrm{E}(K_0) = \frac{\Gamma(k-1,\mu)(k-1) - \Gamma(k)}{\Gamma(k,\mu) - \Gamma(k)}\,\mu$, with $\Gamma(k) = (k-1)!$.

3.3 Survival of Nearest Neighbor

Let us regard a surviving node N_0 and its nearest neighbor N_1 — the node that has the smallest distance to N_0 in $\mathcal{H}$. Our goal is to derive the probability that the nearest neighbor of a surviving node also survives, i.e., we are interested in the conditional probability $\mathrm{P}(N_1\,\text{survives} \mid N_0\,\text{survives})$.

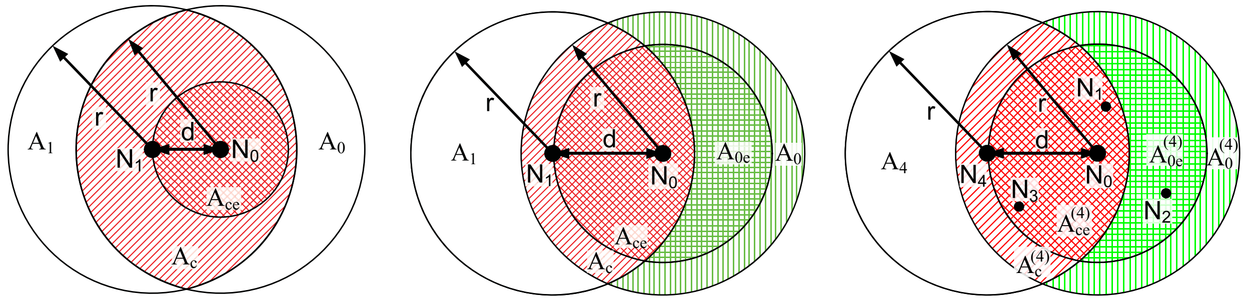
3.3.1 Number of Common Neighbors

Node N_0 has K_0 neighbors in $\mathcal{H}$. Some of these K_0 neighbors are located in the area A_c (see Fig. 3(a)). The probability for an arbitrary chosen neighbor of N_0 to be such a common neighbor of N_0 and N_1 is called p_c. The number of common neighbors is denoted by K_c. If we assume that K_0 is known, K_c follows a binomial distribution $\mathrm{P}\,(K_c = j \mid K_0 = i) = B_{i,p_c}(j) := \binom{i}{j}\, p_c^j\ (1-p_c)^{i-j}$. The unconditional probability $\mathrm{P}(K_c = j)$ is then a weighted sum over all possible values of K_0. Knowing that N_0 survives the thinning ($K_0 \geq k$), we have

$$\begin{aligned}\mathrm{P}\,(K_c = j \mid N_0\,\text{survives}) \;&=\; \mathrm{P}\,(K_c = j \mid K_0 \geq k)\\ =\; &\sum_{i=k}^{\infty} B_{i,p_c}(j)\ \mathrm{P}(K_0 = i \mid N_0\,\text{survives})\,.\end{aligned} \tag{5}$$

Since N_1 is the nearest neighbor of N_0, there are no nodes within a circle of radius d around N_0. If $d \leq \frac{r}{2}$ this "empty circle" lies completely within A_c and is called A_{ce} (see Fig. 3(a)). Otherwise, if $d > \frac{r}{2}$ it lies partly in A_c and partly in A_0, where the two parts are called A_{ce} and A_{0e}, respectively (see Fig. 3(b)). Note that A_c and A_0 still denote the total areas as in Fig. 3(a). Given these definitions, a neighbor of N_0 lies in A_c with a probability

$$p_c = \frac{A_c - A_{ce}}{(r^2 - d^2)\,\pi}\,. \tag{6}$$



(a) Empty neighborhood circle in common area

(b) Empty neighborhood circle exceeding common area

(c) Neighborhood and common areas of N_0 and N_4

Figure 3: Neighborhoods and common areas

3.3.2 Nearest Neighbor Survival Probability

Node N_1 survives the thinning if $K_1 \geq k$. Knowing that it already has $K_c + 1$ neighbors in A_c (namely, N_0 and the common neighbors), it will survive the thinning if at least $k - (K_c + 1)$ further nodes are located in A_1. The number of these non-common neighbors is denoted by $K_1' = K_1 - (K_c + 1)$. As K_1' is independent of K_0, it follows a Poisson distribution with parameter $\rho\, A_1$. Hence,

$$\mathrm{P}(K_1' \geq n) = 1 - \frac{\Gamma\,(n, \rho A_1)}{(n-1)!}\,. \tag{7}$$

The desired survival probability $\mathrm{P}(N_1\,\text{survives} \mid N_0\,\text{survives})$ can now be written as

$$\begin{aligned} &\mathrm{P}(K_1' + K_c + 1 \geq k \mid K_0 \geq k) = \\ &= \sum_{i=k}^{\infty} \left(\sum_{j=0}^{i-1} \mathrm{P}(K_1' \geq k - j - 1)\, B_{i,p_c}(j) \right) \\ &\qquad \cdot \mathrm{P}(K_0 = i \mid N_0\,\text{survives}) \;=:\; p_1(d)\,, \end{aligned} \tag{8}$$

where the probabilities in the sum are given by (5) and (7), respectively. This expression is a function of the node density ρ, the thinning parameters k and r, and the distance d.

The distance between the two nodes is in general not known but is a random variable D. For a homogeneous Poisson process, the pdf of the nearest neighbor distance is $f_D(d) = 2\pi\rho\, d\, \mathrm{e}^{-\rho d^2 \pi}$ [1]. Since we know that $d \leq r$ we have to normalize this term and get $f_D(d \mid d \leq r) = \frac{f_D(d)}{\int_0^r f_D(d)\,\mathrm{d}d}$. This leads to the overall survival probability

$$\mathrm{P}(N_1\,\text{survives} \mid N_0\,\text{survives}) = \int_0^r p_1(d)\; f_D(d \mid d \leq r)\,\mathrm{d}d\,.$$

3.3.3 Calculation of Areas

To compute the above values, we require the size of the different areas. The common area is $A_c = 2r^2 \cdot \arccos(d/2r) - d \cdot \sqrt{r^2 - d^2/4}$. The area A_1 can be computed with $A_1 = r^2\pi - A_c$. The "empty area" A_{ce} is part of A_c; it is

$$A_{ce} = \begin{cases} d^2\pi & \text{if } d \leq \frac{r}{2} \\ d^2 \cdot \arccos\left(1 - \frac{r^2}{2d^2}\right) + \\ \quad + r^2 \cdot \arccos\left(\frac{r}{2d}\right) - \frac{r}{2}\sqrt{4d^2 - r^2} & \text{else.} \end{cases} \tag{9}$$

3.4 Survival of Other Neighbors

We generalize the result of the last section and derive the survival probability $p_l := \mathrm{P}(N_l\,\text{survives} \mid N_0\,\text{survives})$ for the lth nearest neighbor N_l with $l = 1, 2, \ldots, k$, i.e., the node with the lth smallest distance to N_0. In the derivation of p_1 a circle of radius $d = d_1 = d(N_0, N_1)$ around N_0 is empty. For the derivation of p_l with $l > 1$ exactly $l - 1$ neighbors are located within a circle of radius $d_l := d(N_0, N_l)$, namely $N_1, N_2, \ldots, N_{l-1}$. Such a circle is called C_l and can be divided into two parts: $A_{ce}^{(l)}$ and $A_{0e}^{(l)}$ (see Fig. 3(c) where $l = 4$). The nodes inside $A_{ce}^{(l)}$ are common neighbors of N_0 and N_l.

Here we have to distinguish two cases: If $d_l \leq \frac{r}{2}$ the area A_{0e} equals zero and therefore all these $l - 1$ nodes are inside $A_{ce}^{(l)}$. Otherwise the number of nodes within $A_{ce}^{(l)}$ is binomially distributed with $B_{l-1,p_{ce}^{(l)}}$. The probability that a given node in C_l is located in $A_{ce}^{(l)}$ is

$$p_{ce}^{(l)} = \begin{cases} 1 & \text{if } d_l \leq \frac{r}{2} \\ \frac{A_{ce}^{(l)}}{d_l^2 \pi} & \text{else.} \end{cases} \tag{10}$$

The neighbors of N_0 that are further away than N_l are located outside C_l. The probability that such a node is located within $A_c^{(l)} \setminus A_{ce}^{(l)}$ is

$$p_c^{(l)} := \frac{A_c^{(l)} - A_{ce}^{(l)}}{(r^2 - d_l^2)\pi}. \tag{11}$$

Therefore, the number of nodes within $A_c^{(l)} \setminus A_{ce}^{(l)}$ is binomially distributed with $B_{K_0-l,p_c^{(l)}}$.

If N_0 and N_l have j common neighbors, some of them are in $A_{ce}^{(l)}$ and all others are in $A_c^{(l)} \setminus A_{ce}^{(l)}$. Therefore the number of common neighbors is now a combination of these two binomial distributions, and we get (compare with (5))

$$\begin{aligned} &P(K_c^{(l)} = j \mid N_0\text{ survives}) \\ &= \sum_{i=k}^{\infty} \left(\sum_{h=0}^{j} B_{i-l,p_c^{(l)}}(h) \cdot B_{l-1,p_{ce}^{(l)}}(j-h) \right) \\ &\qquad \cdot \mathrm{P}(K_0 = i \mid N_0\text{ survives}). \end{aligned} \tag{12}$$

Next, we derive the survival probability p_l of the node N_l under the condition that N_0 survives and get

$$\begin{aligned} p_l \;=\; &\sum_{i=k}^{\infty} \left(\sum_{j=0}^{i-1} \left(1 - \frac{\Gamma(k-j-1, \rho \cdot A_1)}{(k-j-2)!}\right) \right. \\ &\left. \cdot \sum_{h=0}^{j} B_{i-l,p_c^{(l)}}(h) \cdot B_{l-1,p_{ce}^{(l)}}(j-h) \right) \\ &\cdot \mathrm{P}(K_0 = i \mid N_0\text{ survives}). \end{aligned}$$

3.5 Nearest Neighbor Distance

Finally, we derive the pdf of the distance between a node and its nearest neighbor in $\mathcal{I}$. Let again N_0 be a node that survives the thinning. The pdf $f_{D_l}(d)$ of the distances D_l to the lth neighbor N_l ($l \in \mathbb{N}$) **before thinning** is [5]

$$f_{D_l}(d) = \frac{2\pi^l \rho^l d^{2l-1}}{(l-1)!}\, \mathrm{e}^{-\pi\rho d^2}\,. \tag{13}$$

If the nearest neighbor N_1 of a surviving node N_0 also survives the thinning, the nearest neighbor distance stays the same as before thinning. If N_1 does not survive, there are two options: node N_2 survives or is removed. If it survives, it is the new nearest neighbor after thinning; therefore the nearest neighbor distance corresponds to $f_{D_2}(d)$. This logic can be continued further for higher l.

Summing over all possible new nearest neighbors with respect to their probabilities, the pdf of the nearest neighbor distance **after thinning** is

$$f_{D'}(d) = \sum_{l=1}^{\infty} p_l f_{D_l}(d) \prod_{k=1}^{l-1} q_k. \tag{14}$$

Since this infinite sum is hard to compute, it is important that the product $\prod_{k=1}^{l-1} q_k$ gets smaller for larger l. Therefore it is reasonable to do an approximation of (14) by stopping the summation at a certain index $l = n$ (e.g., where this product is below a certain threshold ϵ), i.e.,

$$f_{D'}(d) \approx f_n(d) \prod_{j=1}^{n-1} q_j \; + \sum_{l=1}^{n-1} p_l f_{D_l}(d) \prod_{k=1}^{l-1} q_k. \tag{15}$$

Comprehensive simulations have shown that this approximation is sufficiently accurate for reasonably chosen ϵ.

Fig. 4 depicts the pdf of the nearest neighbor distance before thinning (red) and after thinning (black). We use $r = 2$, $\rho = 2.5$, and three different values of k. For $k = 17$ (left), more than 99% of the initial nodes survive. Thus there is almost no change in the pdf. As k increases, the number of nodes surviving decreases and therefore the nearest neighbor distance increases. For $k = 27$ (middle) there are about 80% remaining, and at $k = 37$ (right) only 10% survive.

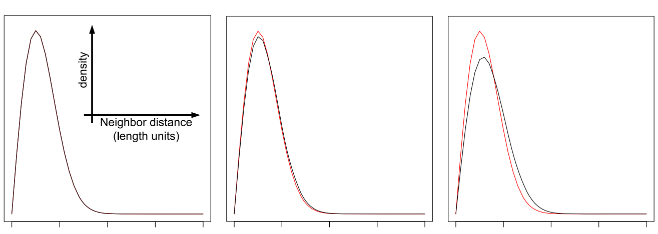
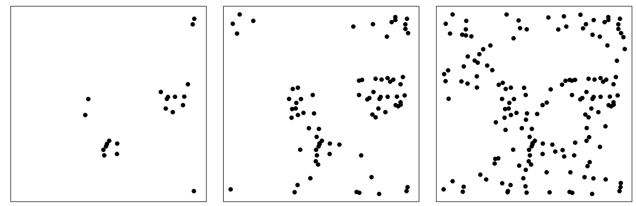


Figure 4: Pdfs of D and D'

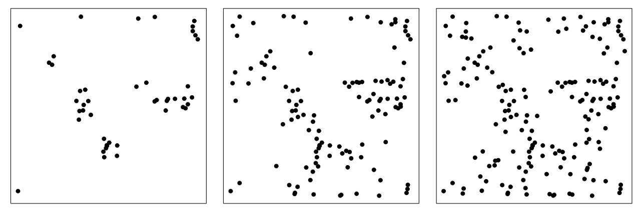
4. VISUALIZATION

This section shows, by example, what typical spatial distributions $\mathcal{I}$ look like and how the input parameters impact its shape. We base the simulations on a uniform random distribution of $m = 200$ nodes on an $a \times a$ square with $a = 5$. The original node density of a corresponding Poisson process is $\rho = m/a^2 = 8$. Thinning is applied without border effects.

We first analyze the **impact of the thinning parameters** on the node distribution. In a first series of experiments thinning is performed with a varying neighborhood radius r. For $k = 10$, Fig. 5(a) shows how the node distribution changes if we increase r from 0.5 (left) over 0.6 (middle) to 0.7 (right). Fig. 5(b) depicts the same experiment with $k = 6$ and $r = 0.4$ (left) over 0.5 (middle) to 0.6 (right).



(a) $k = 10$



(b) $k = 6$

Figure 5: Simulations with fixed k and increasing r

In a second series of experiments, the neighborhood radius is kept constant to $r = 0.5$, while k is increased stepwise. Fig. 6 gives example distributions with stepwise increment of k from $k = 3$ (top left) to 10 (bottom right).

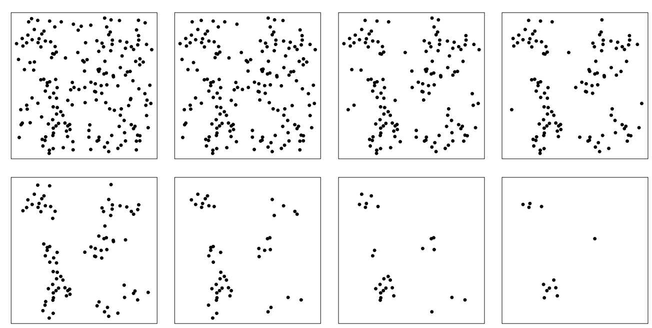


Figure 6: Simulations with fixed r and increasing k

In a third series, thinning is done with correlated values of r and k. For given r, the *required* number of neighbors is always chosen to be the *expected* number of neighbors, i.e., $k = \lfloor \mu \rfloor = \left\lfloor \frac{m\,\pi}{a^2}\, r^2 \right\rfloor$. Fig. 7 depicts three example distributions $\mathcal{I}$ after thinning. Many small clusters result from a thinning with small r and small k, while the number of clusters decreases with increasing r and k. The expected number of nodes remaining for the given parameters are $\mathrm{E}(m') = 125$, 112, and 107, respectively. The actual number of nodes remaining are $m' = 117$, 114, and 118.

Thinning can be performed several times on the same set of nodes. After some number of iterations, such **multiple thinning** converges, either because all nodes are removed or the remaining nodes are located very close together. The effect of multiple thinning is depicted in Fig. 8 (same parameters as above and $k = 6$) with convergence after 6 iterations ($m' = 23$). After the first thinning the clusters are still blurred; after a few iterations they become clearer; finally

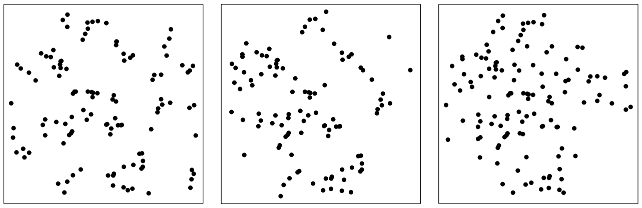


Figure 7: Simulations with increasing r and k

each node has at least k nodes in its neighborhood, and there are no standalone nodes left. Each step but the final one removes nodes that are "important" to the remaining nodes, i.e., they survived only due to the presence of those nodes.

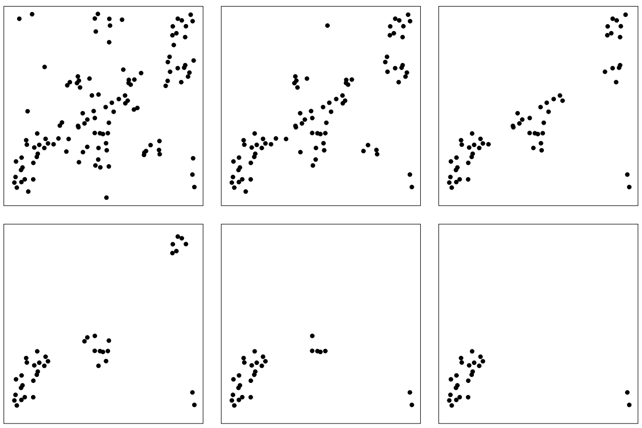


Figure 8: Multiple thinning

5. RELATED WORK

Avidor and Mukherjee [6] introduce a model in which "clump centers" are distributed with a homogeneous Poisson process. For each center, a random number of nodes is chosen, and they are distributed uniformly over a disk of a certain radius. This model is also used in [7] to study coverage and outage probabilities in hybrid ad hoc networks. A similar approach is employed by Vilzmann *et al.* [8] with Gaussian distributed nodes around the clump centers. Basu and Redi use a similar method to investigate the energy efficiency of packet overhearing in sensor networks [9]. Such type of models can be classified as "Poisson cluster processes" [1]. One of their disadvantages is that the stochastic properties are difficult to compute, because the clusters can randomly overlap.

Two alternative models are presented in [10]: the first model uses a Gaussian deviation from ideal grid points; the second model retrieves a subset of nodes from a Poisson process, selecting those nodes that are closest to grid points.

Further papers address the impact of the node distribution on the performance of wireless systems. Baier and Bandelow [11] realize that theoretical computations of the capacity (based on uniform distributions) overestimate the capacity in a real-world scenario. Adrian et al. [12] analyze the effects of different non-uniform distributions on CDMA systems.

Finally, note that the books [1,2] mainly use the term "thinning" to remove nodes *independently* from other nodes; the resulting node locations are again *uniformly* distributed. An alternative to thinning is the class of doubly stochastic processes — called Cox processes [1, 13]. It basically randomizes the density measure ρ of a Poisson process, hence being a generalization of a Poisson process.

6. CONCLUSIONS

We presented a method to generate an inhomogeneous node distribution for simulations of wireless systems. The method is based on a neighborhood-dependent thinning of nodes with two parameters k and r. We analyzed the impact of k and r on the resulting distribution and derived equations for important stochastic properties.

Our goal was to provide fellow researchers with a simple method to create an inhomogeneous distribution with well-known stochastic properties. We hope that the paper stimulates researchers to study the performance of protocols under inhomogeneous conditions and to compare the results with those of homogeneous distributions. Interested readers are invited to request the source code of the model (written in *R*) from the authors.

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